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Arrays of three-dimensional wave-energy absorbers

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The behaviour of a single linear array of five equally spaced semi-immersed spheres, absorbing energy in a single mode from a regular wave train, is studied both for optimal tuning and for constrained body displacement amplitudes. This is extended to consideration of two parallel rows of such devices. Finally, the spheres are replaced by identical bodies of a particular geometry, containing a strong angular variation, which are studied using a thin-ship approximation.

1. Introduction

Recently, Evans (1979) and Falnes (1980) have independently provided a general formulation for the optimal power absorption characteristics of an array of interacting wave power devices. Both authors illustrated the theory by considering a single linear array of small identical heaving buoys, or 'point absorbers', for various angles of incidence of the waves to the array; the q-factor, which describes the strength of the mean interaction between members of the array, was then plotted as a function of the (equal) spacing between the bodies. These 'point absorbers' can be defined as each possessing a vertical axis of symmetry and being such that the wave field created by the motion of any one of them is not influenced by the presence of the remaining array members. An important property of an array of identical point absorbers is that the precise device geometry is not required in order to compute q.

This paper considers the implications of the theory in more detail. If the body geometry is not specified then the body displacements cannot be predicted even though the optimal absorption characteristics are known. This is important from theoretical and practical considerations, since too large a body displacement violates the linear theory and imposes severe constraints on the device design, via the power take-off mechanisms. In § 3 the body geometry is chosen to be a semi-immersed sphere, for which the far-field wave amplitude due to a single heaving body has been calculated by Havelock (1955). Regarding the spheres as point absorbers enables their optimal wave-absorbing properties and their optimal displacements to be determined.

The point absorbers contain no horizontal angular variation in their body geometry. In § 4 thin wedge-shaped bodies are considered, absorbing energy from heave oscillations; these have been introduced previously by Evans (1979). Their geometry exhibits considerable angular variation and is used as a model to study the influence of body shape in array absorption characteristics. Furthermore, they have the advantage of permitting an explicit expression for the exciting force, dependent solely upon the incident wave potential, because of the 'thin-ship' assumption.

2. Theory

We consider an array of N absorbers constrained to make small oscillations in one mode of motion and of radian frequency ω in response to a regular incident wave train of elevation $A\cos(\kappa x\cos\beta+\kappa y\sin\beta+\omega t)$ in water of infinite depth. In this co-ordinate system the horizontal Oxy plane is coincident with the mean water level and the z axis is positive in the vertically upwards direction. The angle β is defined such that the incident wave train makes an angle $\pi+\beta$ with the positive x axis.

The power which such a system can absorb has been shown by Evans (1979) to be

$$P(\beta) = \tfrac{1}{8}\mathbf{X}_s^*\mathbf{B}^{-1}\mathbf{X}_s - \tfrac{1}{2}(\mathbf{U}_0-\tfrac{1}{2}\mathbf{B}^{-1}\mathbf{X}_s)^*\,\mathbf{B}(\mathbf{U}_0-\tfrac{1}{2}\mathbf{B}^{-1}\mathbf{X}_s). \tag{2.1}$$

A derivation of this result is given in the appendix. Here $\mathbf{X}_s$ and $\mathbf{U}_0$ are column vectors representing the complex time-independent component of the exciting force and body velocity respectively, i.e. the exciting force on the mth body due to the incident wave train when all bodies are held fixed and the velocity of the mth body when the system is allowed to respond to the incident wave are given by $\mathrm{Re}\{X_{sm}(\beta)\,e^{i\omega t}\}$ and $\mathrm{Re}\{U_{0m}(\beta)\,e^{i\omega t}\}$ respectively. In equation (2.1) $*$ denotes the conjugate transpose.

The $N\times N$ real symmetric matrix $\mathbf{B}$ is the usual radiation-damping matrix and, by application of Green's theorem and the method of stationary phase, it can be shown that the matrix $\mathbf{B}$ is related to the exciting forces in deep water by

$$B_{mn} = \frac{1}{8\lambda P_w}\int_0^{2\pi} X_{sm}(\theta)\,\bar{X}_{sn}(\theta)\,d\theta, \tag{2.2}$$

where λ is the wavelength of the incident wave, $P_w = \rho g^2A^2/4\omega$ is the power per unit frontage of the incident wave and the overbar on $X_{sn}(\theta)$ denotes the usual complex conjugate. This result for N bodies in one mode of motion has been given by Srokosz (1979) and a brief outline of the derivation is given in the appendix. It is assumed here that $\mathbf{B}^{-1}$ exists; discussion of the validity of this assumption is contained in both Evans (1979) and Falnes (1980).

The expression for the power absorption (2.1) can be maximized with respect to the body velocity to obtain

$$P_{\max} = \tfrac{1}{8}\mathbf{X}_s^*\,\mathbf{B}^{-1}\mathbf{X}_s, \tag{2.3}$$

which occurs when $\mathbf{U}_0 = \frac{1}{2}\mathbf{B}^{-1}\mathbf{X}_s$. This maximization procedure involves the tuning of both the amplitude and phase of each of the body oscillations to the quantities specified above in terms of the exciting forces on each of the array members.

A further important quantity is the absorption length l_{abs}, defined as the width of a two-dimensional wave train having the same mean power as the body extracts:

$$l_{\mathrm{abs}} = P/P_w. \tag{2.4}$$

In particular, when the system is performing optimally we have $l_{\mathrm{abs}} = l_{\max}$ which is defined by

$$l_{\max} = \frac{P_{\max}}{P_w}. \tag{2.5a}$$

This can be written

$$l_{\max}(\beta) = \frac{\lambda}{2\pi}N\,q(\beta) \tag{2.5b}$$

for heave motions. The quantity $\lambda/2\pi$ is just the maximum absorption length for a single heaving point absorber (λ/π for surge or sway modes) so that $q(\beta)$ represents the mean gain factor for each member of an interacting system of N members as compared with the absorption length for a single point absorber.

The exciting forces on each of the array members are related to the corresponding radiation problem. If the mth body alone is forced to oscillate at radian frequency ω with unit velocity amplitude and all other bodies are held fixed, then the potential function describing the motion has the following asymptotic behaviour, in cylindrical polar co-ordinates, at large distances from the body:

$$\Phi_m(R,\theta,z,t) \sim \mathrm{Re}\{g_m(\theta)\,R^{-\frac{1}{2}}\,\mathrm{e}^{\kappa z-i\kappa R+i\omega t}\}. \tag{2.6a}$$

The quantity $g_m(\theta)$ must contain both the angular variation of the wave amplitude and a phase factor due to the mth body not being positioned at the origin. These two characteristics can be separated by writing $g_m(\theta)$ in the following way:

$$g_m(\theta) = f_m(\theta)\,\kappa^{\frac{1}{2}}\exp\left[i\kappa l_m\cos\left(\beta-\alpha_m\right)\right], \tag{2.6b}$$

where l_m and α_m are the distance from the origin and the angular displacement measured from the x axis respectively and $f_m(\theta)$ is usually referred to as the far-field amplitude. For one body in one mode of motion, the exciting force was first shown to be related to the far-field amplitude by Newman (1976). Here we give the extension of that result for N bodies in one mode of motion due to Srokosz (1979):

$$X_{sm}(\beta) = \frac{\rho g A}{\kappa}\sqrt{(2\pi)}\,e^{-\frac{3}{4}i\pi}f_m(\beta)\exp\left[i\kappa l_m\cos\left(\beta-\alpha_m\right)\right], \tag{2.7}$$

which is obtained on substitution from (2.6*b*) into equation (A 2) of the appendix.

Accordingly, to solve the optimal problem completely it is necessary to know either the far-field amplitudes or the exciting forces. In particular, from (2.2), (2.3) and (2.5*b*) we have

$$q(\beta) = X^*_{sm}(\beta)\left[\frac{1}{2\pi}\int_0^{2\pi}X_{si}(\theta)\,\bar{X}_{sj}(\theta)\,d\theta\right]^{-1}_{mn}X_{sn}(\beta), \tag{2.8a}$$

where repeated suffixes denote summation and where $[\;]^{-1}_{mn}$ denotes the (m,n)th term of the inverse of the matrix whose (i,j)th term is given in the $[\;]$. For an array of identical bodies which produce little or no diffracted wave field, then it is readily seen that only the angular dependence contained in $X_{sm}(\beta)$, or alternatively $f_m(\beta)$ is required to calculate $q(\beta)$ or $l_{\mathrm{abs}}(\beta)$. This property has already been employed by Evans (1979) in the study of point absorbers and thin ships. However, if the corresponding body displacements are required, then these are given by the non-dimensional column vector $\mathbf{D}$, such that the displacement of the mth body is $\mathrm{Re}\{AD_m e^{i\omega t}\}$, which at optimal tuning is

$$\mathbf{D} = -\frac{i}{2\omega A}\mathbf{B}^{-1}\mathbf{X}_s \tag{2.8b}$$

and this is clearly dependent on the body geometry properties contained in $\mathbf{X}_s$.

For an array of heaving point absorbers, i.e. bodies of revolution oscillating vertically along their axes, the consequences of (2.8*a*, *b*) are clear: while $q(\beta)$ remains largely independent of body size, this is not so for the body displacements and the linearizing assumptions of the theory may well become invalid for certain body sizes and wave frequencies.

Large body displacements are also undesirable from a physical viewpoint since they introduce practical problems in device construction and operation. Consequently it may become necessary to constrain the body displacements so that they do not exceed a specified multiple of the incident wave amplitude and this can be done by damping the body motions accordingly. If the absorption length l_{abs} is written in terms of $\mathbf{X}$ and $\mathbf{D}$ ($\mathbf{D} = -i\mathbf{U}_0/\omega A$) then from (2.1) and (2.4) we have

$$l_{\text{abs}} = \frac{i\kappa}{\rho g A}[\mathbf{X}_s^*\mathbf{D} - \mathbf{D}^*\mathbf{X}_s] - \frac{2\omega\kappa}{\rho g}\mathbf{D}^*\mathbf{B}\mathbf{D}, \tag{2.9}$$

where the body motions are still assumed to have an $e^{i\omega t}$ time dependence. The quantity l_{abs}, for known $\mathbf{X}_s$, can be regarded as a function of the $2N$ variables comprising the amplitude and phase of each body displacement. This expression can be maximized numerically subject to the constraint that the body displacement amplitudes must not exceed a specified value, and the appropriate values of l_{abs}, amplitudes and phases can be found. These may then be compared with those obtained at optimal tuning and, of course, the values will coincide if the optimally tuned body displacement amplitudes are all smaller than the value specified by the constraint.

The theory given above is applicable to an array of N bodies in only one mode of motion, but there is no intrinsic difficulty in extending the theory to describe N bodies in M modes of motion, for any value of $M > 1$ provided each mode is capable of independently absorbing energy from the waves.

In order to use the preceding theory it is necessary to know either X_{sm} or $f_m(\theta)$ for each m. This is a difficult problem for an array of arbitrary body shapes and analytic solutions are only known for single bodies with particular simple geometries. Accordingly, we use the approximation, introduced by Evans (1979), that the angular function $f_m(\theta)$ does not depend on the presence of the other bodies; this may be equivalently stated that the bodies are small enough not to produce a significant diffracted wave field. Furthermore by consideration only of arrays of identical bodies operating in the same mode, the above approximation shows that if $f_m(\theta)$ is known for the mth body then it is known for all the other bodies also. It is felt that this is probably a good approximation provided that the bodies are not too close together.

3. Arrays of heaving semi-immersed spheres

The simplest body geometry for which the far-field amplitude $f(\theta)$ is known analytically is for a single heaving semi-immersed sphere. The solution has been given by Havelock (1955) and in the co-ordinate system used here $f(\theta)$ takes the form

$$f(\theta) = \kappa a^2(2\pi)^{\frac{1}{2}}\,\Omega\, e^{-\frac{1}{4}i\pi + i\chi}, \tag{3.1}$$

where κ is the wavenumber of the generated wave train and a is the radius of the sphere. As expected, the quantity $f(\theta)$ is independent of θ.

The constants Ω and χ which appear in (3.1) are both real and dependent upon κa. In terms of the constants C and D used by Havelock Ω and χ are defined by

$$\Omega e^{i\chi} = C - iD. \tag{3.2}$$

The constants C and D can be regarded as known functions of κa so that Ω and χ are also known everywhere.

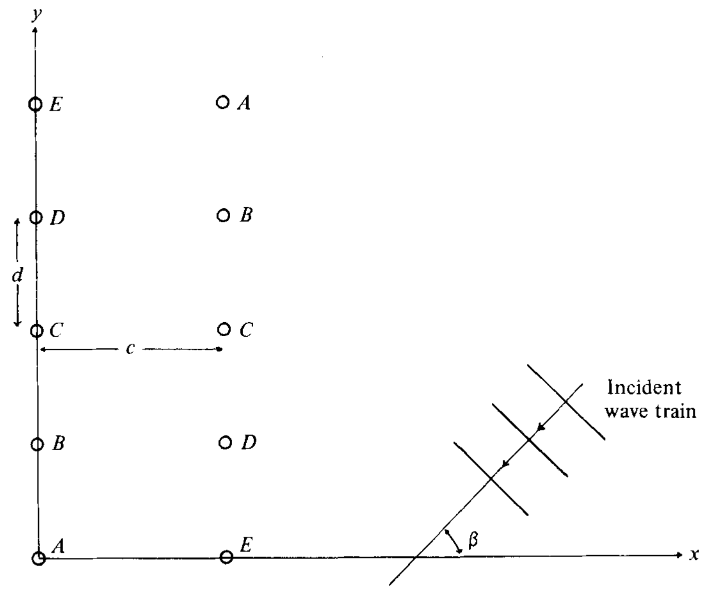

FIGURE 1. Plan of double-row array subject to incident wave train with arbitrary angle of incidence β. Bodies indicated by the same letter undergo the same displacement amplitude at optimal tuning. For the special cases of head and beam seas $B = D$ and $A = E$ (symmetry text given in § 3 *b*).

In order to study the influence of the body spacing at a particular wave frequency it is necessary to fix the parameter κa and the value chosen was $\kappa a = 0{\cdot}4$. This value corresponds well with the envisaged dimensions of full scale devices. For example if the incident wavelength is 150 m then the value of the sphere radius is 10 m. Additionally, the value of C and D were only given by Havelock for $\kappa a = 0{\cdot}4$, although this is not an important consideration since Havelock's calculations were repeated to obtain C and D (and hence Ω and χ) in the range $0 \leqslant \kappa a \leqslant 5{\cdot}0$; this data was required so that the variation in the properties of a fixed array could be studied if necessary. For a single semi-immersed body, Evans (1976, § 7) has shown that the large natural buoyancy-restoring force can introduce difficulties when tuning the device to certain prescribed frequencies; this problem is acknowledged but plays no part in the optimal formulation described here.

Two array configurations are considered. The first consists of a row of five equally spaced bodies and the second has two parallel rows of five equally spaced bodies forming a rectangle. These are shown in plan in figure 1. For the single row the bodies lie along the y axis and are numbered such that the first body lies at the origin and the fifth at $y = 4d$.

(*a*) *The single row*

We consider first the simpler case of the single row when the system is optimally tuned. The form of $f(\theta)$ given by (3.1) is substituted into (2.7), (2.8*a*), (2.5*b*) and (2.8*b*) to obtain $q(\beta)$, $l_{\max}(\beta)$ and the appropriate body displacements as functions of κa and κd. With $\kappa a = 0{\cdot}4$, the above quantities were calculated numerically for κd in the range $0{\cdot}8 \leqslant \kappa d \leqslant 10$; the lower limit corresponding to the case where the buoys are touching and the upper limit to the case where the buoy centres are 12·5 sphere diameters apart.

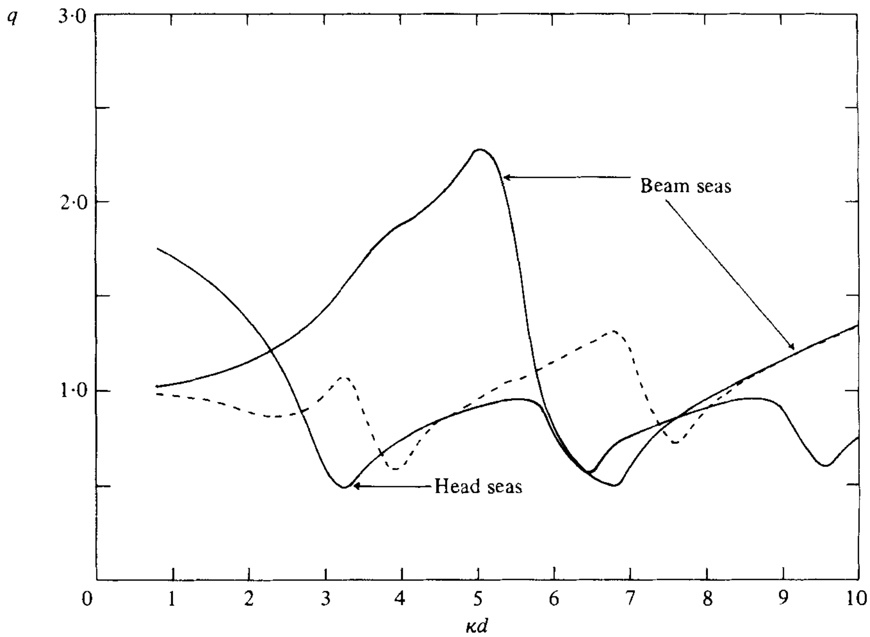

FIGURE 2. Variation of the mean gain factor q with the non-dimensional body spacing κd for an optimally tuned single row of five equally spaced semi-immersed spheres for different values of the angle of incidence β. ——, $\beta = 0$ (beam seas), $\frac{1}{2}\pi$ (head seas); – – –, $\beta = \frac{1}{4}\pi$.

The variation of q with κd is shown in figure 2 for both beam ($\beta = 0$) and head ($\beta = \frac{1}{2}\pi$) seas. At optimal tuning the array is generally more efficient in beam seas than in head seas, the exception being when the bodies are close together, corresponding to $\kappa d < 2{\cdot}65$ or $d/2a < 3{\cdot}31$, where the approximation used to obtain $f_m(\theta)$ from the single body result is least applicable.

For beam seas, q takes a maximum value of about 2·25 at $\kappa d \simeq 5{\cdot}1$, indicating a favourable interaction between members of the array. The minimum value of q is approximately 0·57 occurring at $\kappa d \simeq 6{\cdot}75$ when the bodies are slightly greater than one wavelength apart. This illustrates the critical importance of the body spacing parameter: for the value of κa chosen and for the range of κd used, the maximum and minimum values of q occur within roughly a variation of 2·2 body diameters of each other in the spacing.

In the head seas case, the interaction is usually unfavourable (i.e. $q < 1$) with the behaviour of q appearing to settle to a near-periodic function of κd. The range of q for $\kappa d > 3$ (i.e. *away* from the region where the analysis is least valid) is approximately $0{\cdot}5 < q < 0{\cdot}96$, with the minima occurring when array members are integral multiples of half or full wavelengths apart, i.e. when $\kappa d = n\pi$ for integer n. There does not appear to be an obvious physical analogue for describing the positions of the maxima.

The behaviour of q for an angle of incidence $\beta = \frac{1}{4}\pi$ is also shown in figure 2 (as a broken line). This is included here for completeness to illustrate the transitional stage between the beam and head sea cases. As expected the maximum value of $q(\frac{1}{4}\pi)$ is greater than that of q(head), but less than the maximum of q(beam). Henceforth, we consider only beam and head seas.

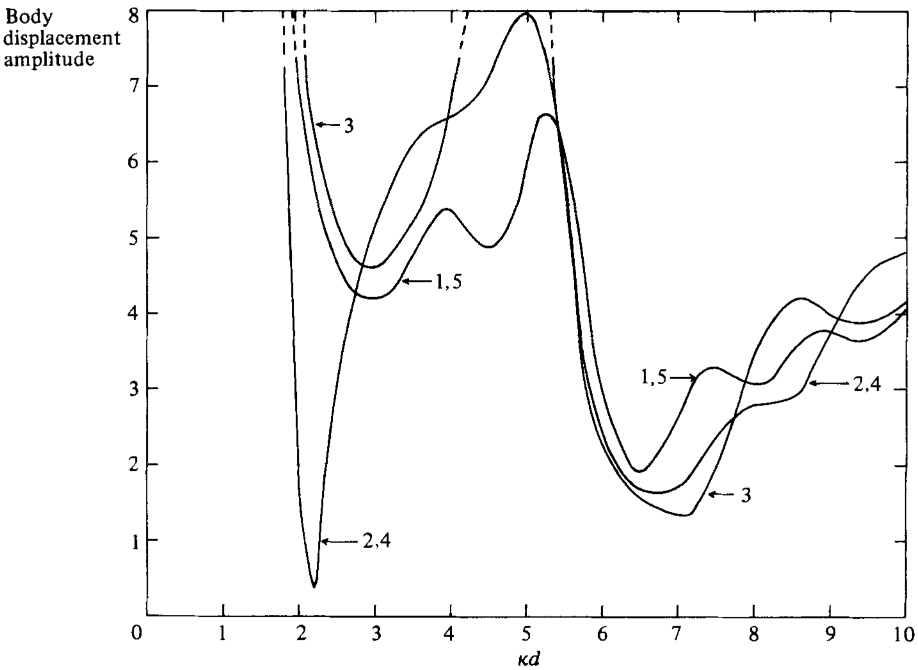

FIGURE 3. Variation of the body displacement amplitudes/incident wave amplitude with body spacing for the optimally tuned single row of five semi-immersed spheres in beam seas. The number next to each curve denotes the appropriate body motion.

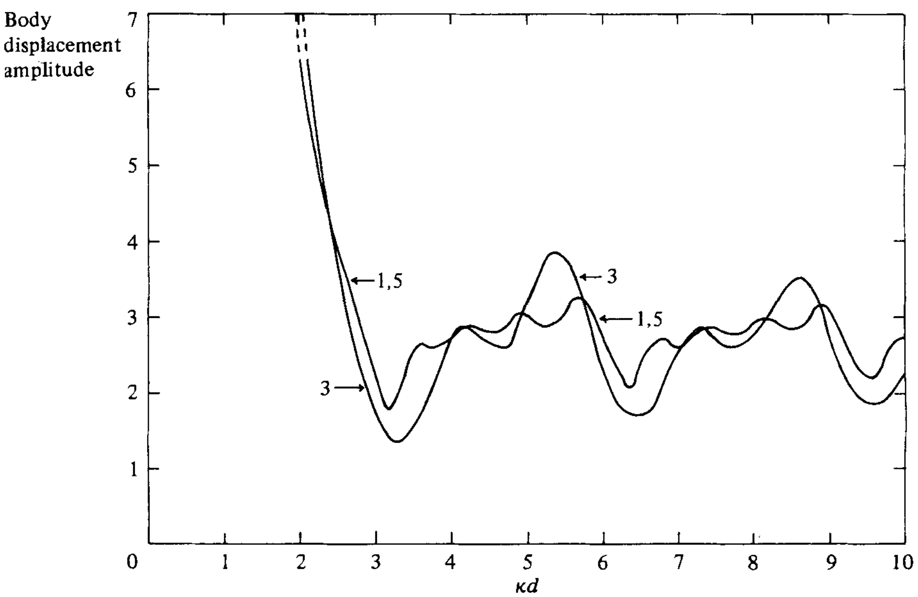

FIGURE 4. As for figure 3, but in head seas.

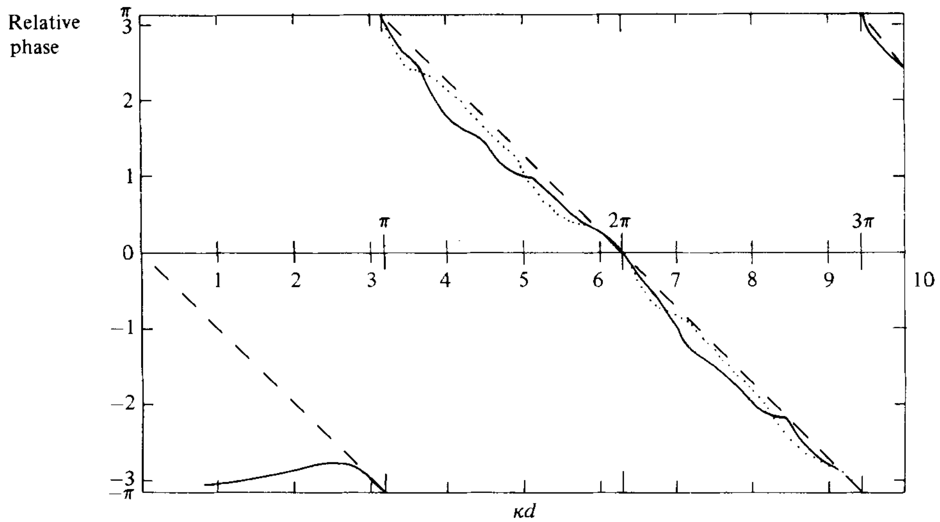

FIGURE 5. Variation of the relative phase, i.e. the phase difference between two adjacent bodies, with the body spacing for the optimally tuned single row of semi-immersed spheres in head seas. ——, the relative phase of the (first and second) and (fourth and fifth) bodies; . . . , the (second and third) and (third and fourth) bodies; – – –, a constant phase difference of $-\kappa d$.

The body displacement amplitudes for beam and head seas are shown in figures 3 and 4 respectively. In each case, since the array operates in an optimal manner, the displacements are symmetric about the mid-point of the array as shown by Evans (1979). For beam seas, the displacement amplitudes are seen to be generally considerably larger than the incident wave amplitude (which has unit amplitude on this scale), so that the favourable interaction shown in figure 2 is achieved with a possible violation of the linearization assumptions. Relative to each other, all the bodies are in phase for $\kappa d > 2{\cdot}1$.

In head seas, figure 4 shows that the displacement amplitudes are typically of the order of two or three times the incident wave amplitude, with the largest amplitudes corresponding to the largest values of q(head) shown in figure 2. The curve for the second and fourth bodies is not shown, simply because it lies very close to the two which are given and would not illustrate any new features of the motions. Intuitively, for small bodies the phases of the array members might be expected to correspond to the phase of the incident wave moving down the array. If is this were so then the phase difference between any two adjacent bodies would be $-\kappa d$ in the direction of wave propagation.

The relative phases between adjacent members of the array are shown in figure 5. From symmetry arguments the phase difference between the (first and second) and (fourth and fifth) bodies must be the same; likewise for the (second and third) and the (third and fourth), but no relationship is suggested to link the first, second and third bodies together. It is seen from figure 5 that for $\kappa d > 3$ the difference in phase between the (first and second) bodies is generally close to $-\kappa d$. The phase difference between the (second and third) bodies is indicated by the dotted line and it too lies close to $-\kappa d$, the differences which occur being due to the interaction mechanism.

It is readily seen from figures 2 and 5 that q(head) takes its minima whenever the

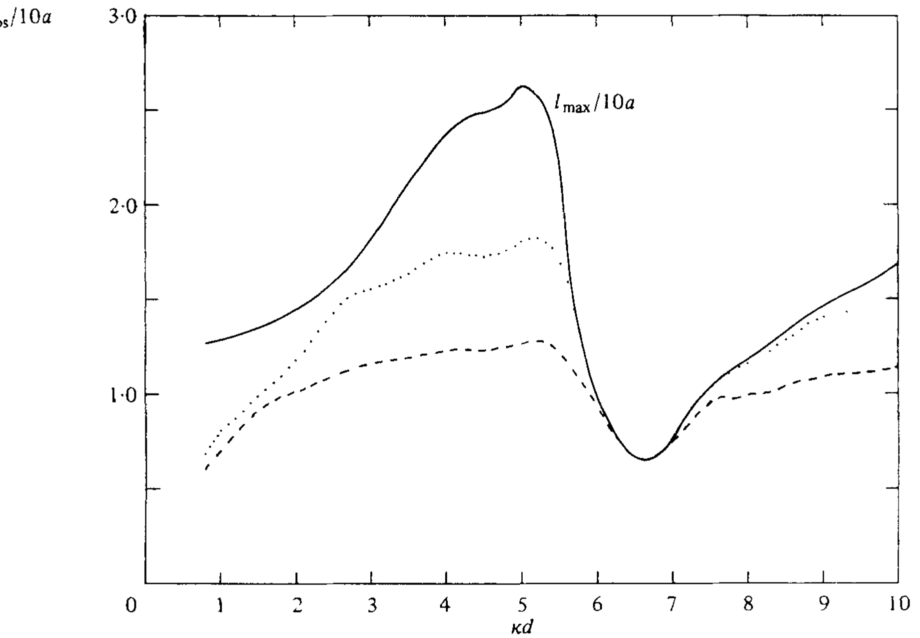

FIGURE 6. Variation of the non-dimensional absorption length $l_{abs}/10a$ with body spacing for the single row of semi-immersed spheres in beam seas. ——, optimal unconstrained motion; . . ., constrained motion, maximum body displacement < three times incident wave amplitude; – – – constrained motion, maximum body displacement < twice incident wave amplitude.

phase difference between adjacent bodies is $-\kappa d$, i.e. when $\kappa d = n\pi$. The importance of the relative phases (and hence the interaction mechanism) depicted in figure 5 has been studied numerically. Preliminary results for different values of κd, with $\kappa d > 3$, suggest that, if all of the relative phases are constrained to be exactly equal to $-\kappa d$, then the corresponding value of the maximum absorption length, obtained from a numerical optimization of equation (2.9), can be as much as 25 % less than the value at optimal tuning.

The large body displacements at optimal tuning in beam seas, shown in figure 3, illustrate the desirability of constraining the body displacements when necessary so that the linear theory is not violated. Further justification for such a procedure has already been given in § 2. Accordingly, using (3.1) and (2.7), we optimize (2.9) numerically, using a standard computer library procedure, subject to the constraint that the displacement amplitudes do not exceed certain prescribed multiples of the incident wave amplitude. The results of such optimizations are shown in figure 6, in which the amplitudes are constrained to be less than two and three times the incident wave amplitude. For comparison with the result at optimal tuning, the curve corresponding to the unconstrained motion is also shown.

The quantity plotted against κd is $l_{abs}/10a$, where l_{abs} is the absorption length as previously defined and $10a$ is the frontage to the incident wave in beam seas of five semi-immersed spheres each of radius a. Using the definitions given in (2.4) and (2.5), it is clear that with $\kappa a = 0{\cdot}4$ the favourable criterion $q > 1$ corresponds to

$$l_{abs}/10a > 1{\cdot}25.$$

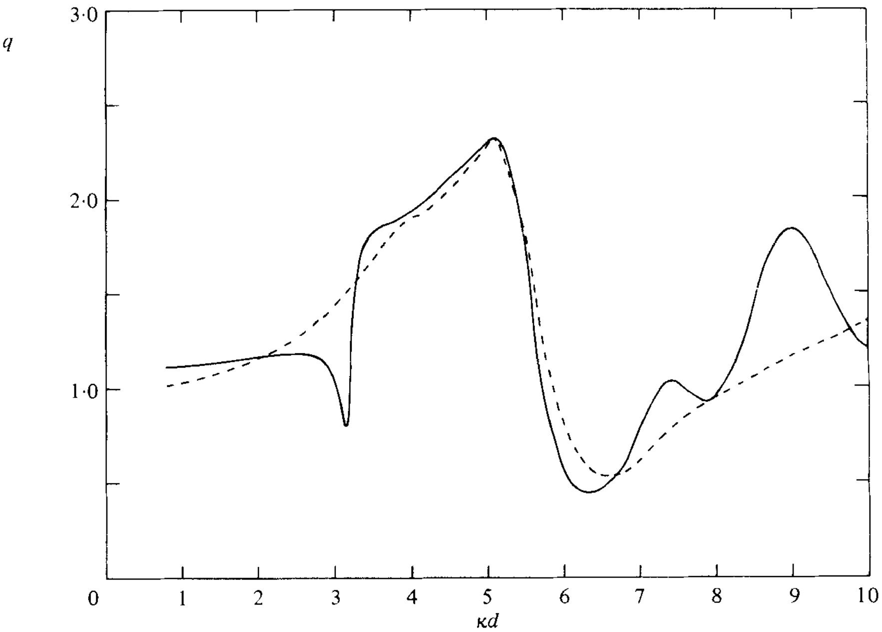

FIGURE 7. Variation of the mean gain factor q with body spacing for an optimally tuned double row and an optimally tuned single row of semi-immersed spheres in beam seas. The spacing between the rows of the double row is equal to the spacing between adjacent members of each row, i.e. $c = d$. ——, double row; - - -, single row.

However, if $1 < l_{abs}/10a < 1{\cdot}25$ then, although the array does not perform collectively as well as the sum of five individual absorbers, it does absorb more power than is incident on the individual members.

Figure 6 shows that if the displacements are constrained to have magnitudes of up to twice the incident wave amplitude then the criterion $q > 1$ is only achieved very near to the optimum spacing $\kappa d \simeq 5$. If a factor of three times the incident wave amplitude is permissible then a curve much nearer the optimum is achieved, although at optimal tuning figure 3 shows the relative amplitudes of the body displacement and the incident waves to be often considerably greater than three. Thus even with constraints of this type the array can perform acceptably well. The phases in the constrained motion are similar to those at optimal tuning, i.e. the bodies move in phase for $\kappa d > 2{\cdot}1$ but not otherwise.

(b) *The double row*

The results presented so far have only been concerned with a one-dimensional array positioned on the y axis, with uniform spacing between the individual bodies. The next step is to consider two-dimensional arrays of the type shown in figure 1, and which have been described previously. Two arrays were studied: in the first the bodies were uniformly spaced in both directions, i.e. $c = d$, and in the second the spacing in the x direction was half that in the y direction, i.e. $c = \frac{1}{2}d$.

The q-factor for an optimally tuned double row with $c = d$ in beam seas is shown in figure 7 and the appropriate curve for a single row from figure 2 is also given so that the two may be compared. The two curves are similar and, although not illustrated,

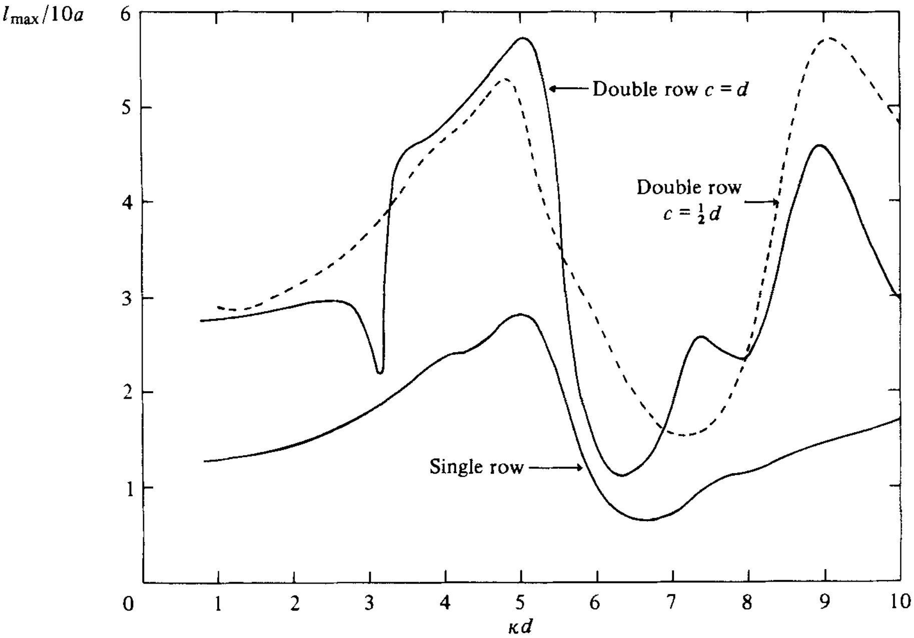

FIGURE 8. Variation of the non-dimensional maximum absorption length $l_{max}/10a$ with body spacing for optimally tuned single and double rows of semi-immersed spheres in beam seas. The double row spacings are $c = d$ and $c = \frac{1}{2}d$ (indicated by a broken line).

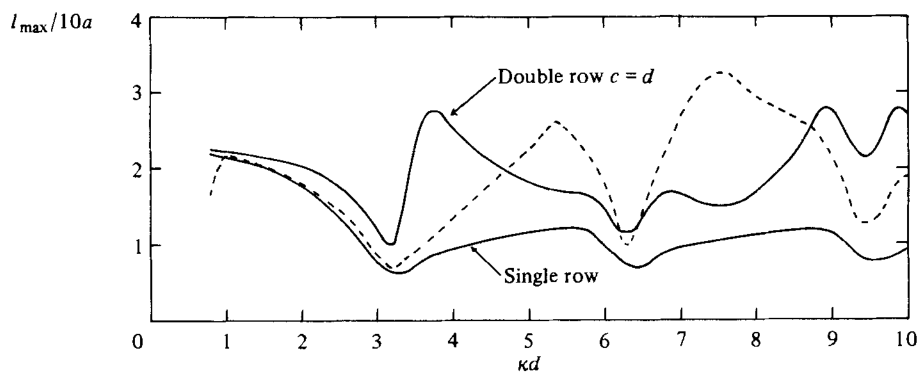

FIGURE 9. As for figure 8, in head seas.

similar behaviour is also exhibited by the corresponding $c = \frac{1}{2}d$ curve. However, the local behaviour of the $c = d$ curve does indicate some interesting features.

There are pronounced local minima when the spacing is either half or one incident wavelength i.e. when $\kappa c = \kappa d = \pi$ or 2π whilst for $\kappa d > 2\pi$ the curve becomes oscillatory. With the exception of the regions surrounding $\kappa d = \pi$ and 2π, the q-factor for the double row is generally marginally better than for the single row, but both curves have approximately the same maximum value of about 2·24 at $\kappa d \simeq 5$. It should be remembered that this implies that the double row extracts double the power, however, as in this case $N = 10$ in (2.5b).

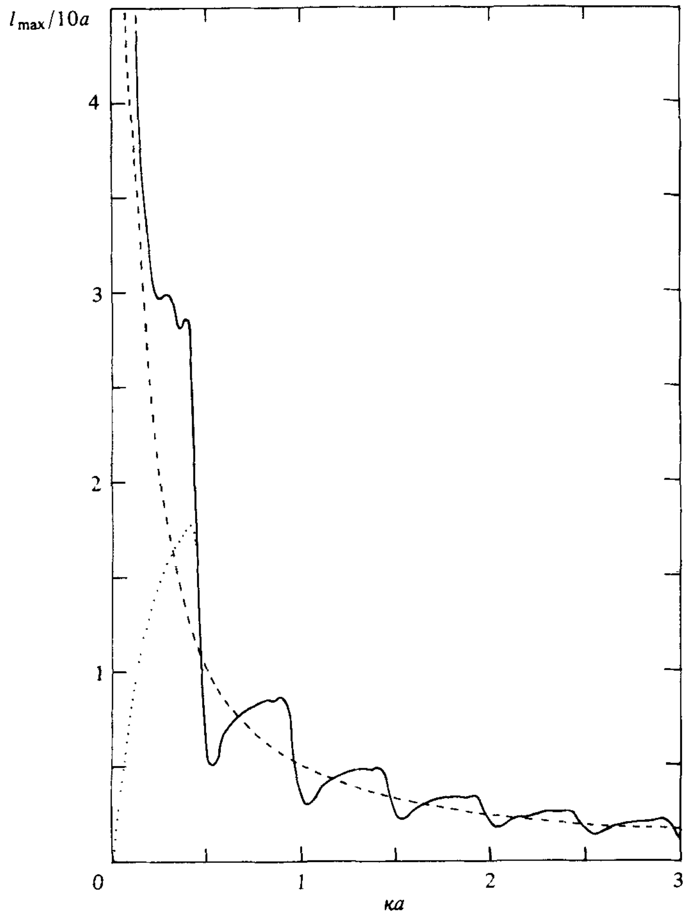

FIGURE 10. Variation of the non-dimensional maximum absorption length $l_{max}/10a$ with frequency κa for an optimally tuned row of five equally spaced semi-immersed spheres in beam seas. The spacing between the bodies is fixed at $d = 12{\cdot}5a$. ——, $l_{max}/10a$ for unconstrained optimal motion; …, $l_{abs}/10a$ for constrained motion, maximum body displacement < three times incident wave amplitude; – – –, mean gain factor $q = 1$.

This is illustrated by considering the non-dimensional maximum absorption length $l_{max}/10a$ for optimally tuned arrays in beam and head seas as shown in figures 8 and 9 respectively. It is readily seen that for both beam and head seas the double rows almost always perform better than the single row, the exception being in head seas for $c = \frac{1}{2}d$ in the physically uninteresting region of $\kappa d < 1$.

The body displacements corresponding to the calculations depicted in figures 8 and 9 have also been obtained. Symmetry arguments analogous to those presented by Evans (1979) must hold for all angles of incidence and the bodies whose amplitudes must be equal for an arbitrary angle of incidence β are shown in figure 10, in which bodies given the same letter have the same displacement amplitude. For the special cases of beam and head seas, further simplication is possible and in both of these cases, in the notation of figure 1, we also have $A = E$ and $B = D$, thus giving only three different amplitude values. The magnitude of the amplitudes follows the same trends as shown for the single row and confirms that the more power absorbed the larger the body displacement

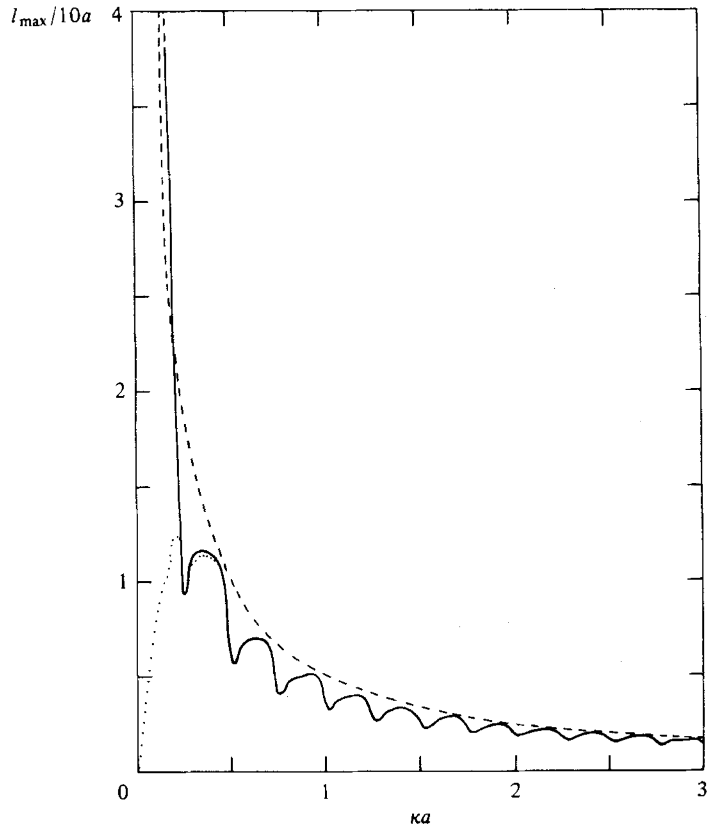

FIGURE 11. As for figure 10, in head seas.

amplitude. It is also possible to constrain the body displacement amplitudes in the way described previously for the single row (as illustrated in figure 6).

The results obtained for the double row subject to the constrained optimization technique, are qualitatively similar (though multiplied by a factor of 2, since there are twice as many bodies) to those for the single row and consequently are not shown.

(c) *The variation of optimal absorption properties with frequency*

In the previous work the incident wavelength and body radius have been kept fixed and only the spacing has been allowed to vary. Equally important is to assess how the optimal absorption properties vary with frequency, for fixed body radius, *and* spacing.

The parameters κa, κd for a, d fixed are both measures of wave frequency; κa will be used here. To determine an appropriate value of d/a we can use the work already presented. When $\kappa a = 0{\cdot}4$, we know from figures 2, 7 and 8 that in beam seas both the single and double arrays perform best when κd is close to 5 and accordingly this value is used for κd. In physical terms if $a = 9{\cdot}55$ m, corresponding to $\kappa a = 0{\cdot}4$ in 150 m waves, then $d/a = 12{\cdot}5$ (i.e. $\kappa d = 5$ when $\kappa a = 0{\cdot}4$) implies that d has a value of approximately 120 m. Although we are only considering optimal motions the choice of $d/a = 12{\cdot}5$ based on the known results for $\kappa a = 0{\cdot}4$ is in effect a partial tuning of the system.

The variation of the maximum absorption length $l_{\max}/10a$ with the frequency parameter, for the single row, in beam and head seas is shown in figures 10 and 11 respectively. The broken line which appears in each figure is the curve corresponding to $q = 1$ and is included so that the strength of the interaction at each frequency can be gauged.

In beam seas the outstanding feature demonstrated in figure 10 is the minima which are achieved whenever $\kappa d = 12 \cdot 5 \times \kappa a = 2n\pi$, for integer n, corresponding to situations where the array members are integral multiples of the incident wavelength apart. A similar phenomenon has been shown to exist by Srokosz (1980), in the study of the behaviour of an infinite array of devices. There are, however, two important differences between our results and those of Srokosz; both are directly attributable to the array having a finite length. Firstly, the minima are non-zero, whereas those of Srokosz are genuine zeros. Secondly, for $\kappa d < 2\pi$ (i.e. $\kappa a < 0 \cdot 5$), the infinite array theory predicts a constant value of $l_{\max}/5d = 0 \cdot 5$ giving, with $d/a = 12 \cdot 5$,

$$l_{\max}/10a = 3 \cdot 125,$$

but this is not so for the finite length array, which suggests that $l_{\max}/10a \to \infty$ as $\kappa a \to 0$.

The head seas result is equally interesting, exhibiting minima whenever the bodies are multiples of half or whole wavelengths apart. A feature of this curve is that the $q = 1$ curve behaves like an upper bound on the $l_{\max}/10a$ curve except for the longest waves, i.e. when $\kappa a < 0 \cdot 22$, corresponding to $\lambda > 270$ m for $a = 9 \cdot 55$ m. This cannot be compared with the infinite array case, since Srokosz's theory is only applicable to beam seas.

The body displacement amplitudes have also been calculated. As a simple illustration of these, we show as dotted lines in figures 10 and 11 the curves for $l_{\mathrm{abs}}/10a$ corresponding to the displacement amplitudes constrained to be less than three times the incident wave amplitude. The $l_{\max}$ and l_{abs} curves, for both the head and beam seas cases, are coincident except for the longest waves where, as expected, the array members experience large displacements to achieve a high level of power absorption.

The optimal curves shown in figures 10 and 11 correspond to a system which is essentially optimally tuned at all frequencies. In practice this is not possible and it would be necessary to tune the system mechanically to one particular frequency. If the tuning was for $a = 9 \cdot 55$ m, $d = 120$ m at incident waves of 150 m, then the optimal value would be attained at $\kappa a = 0 \cdot 4$.

4. An array of heaving thin ships

The semi-immersed spheres studied in § 3 possess a vertical axis of symmetry and consequently the far-field amplitude produced by a single heaving buoy contains no angular dependence. In practice, many of the wave power devices presently undergoing development are not axisymmetric and often exhibit a strong angular variation in the body geometry which will obviously manifest itself in the form of the far-field amplitude.

However, most of the devices under consideration are not readily amenable to simple analytic treatment and calculation of the exciting force or far-field amplitude usually involves detailed numerical work. For more realistic models of these devices

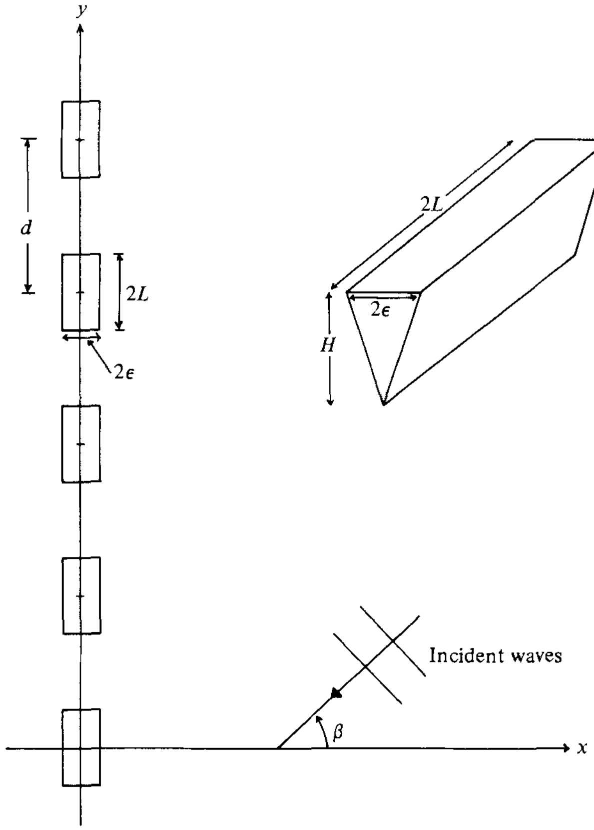

FIGURE 12. Diagrammatic sketch of thin ship and plan of five uniformly spaced thin ships in a single row.

we require a body shape which contains a strong angular dependence and which has a simple analytical representation. An appropriate body is the 'thin ship', absorbing energy through small vertical (heave) oscillations, used previously by Evans (1979).

The 'thin ship' has beam 2ϵ, draft H, and length $2L$. The heave exciting force is obtained by integrating the incident potential over the surface of the body and neglecting the diffracted wave field. This can be justified provided ϵ/H, $\epsilon/L \ll 1$, and $\lambda/L = O(1)$. See Newman (1977, pp. 305–306). A derivation of the exciting force is given in the appendix, where it is shown that

$$X_{sm}(\beta) = -4\rho g A\epsilon L \frac{(1-e^{-\kappa H})}{\kappa H}\left[\frac{\sin(\kappa L \sin\beta)}{\kappa L \sin\beta}\right] e^{i\kappa dm \sin\beta}. \tag{4.1}$$

In computing the maximum power, we choose $\kappa L = 1$ so that in 150 m waves the thin ship has length 48 m which is typical of isolated terminator wave-energy devices such as the Bristol submerged cylinder device (Quarrell 1978) and which also satisfies the condition $\kappa L = O(1)$ required by the thin-ship theory. The other two parameters, ϵ/L and H/L, are constrained by the demands of the thin-ship approximation and accordingly we use the values $\epsilon = L/10$ and $H = L$.

Substitution from (4.1) into (2.2), (2.3) and (2.5a, b), with the values of κL, ϵ/L and

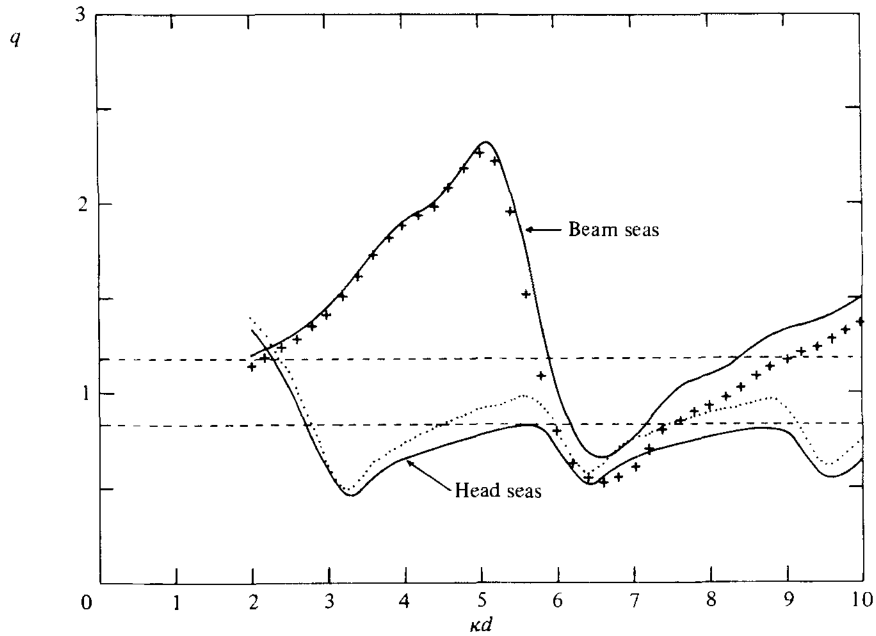

FIGURE 13. Variation of the mean gain factor q with non-dimensional body spacing κd for an optimally tuned, single row of five uniformly spaced thin ships in beam and head seas. + + + (......), the corresponding curves from figure 2 for five semi-immersed spheres in beam (head) seas. – – –, the values corresponding to $q_t = 1$ in beam (the upper line) and head seas.

H/L specified above, enables the q-factor and maximum absorption length to be calculated for any particular array. The results presented here correspond to a single row of five uniformly spaced thin ships in line, analogous to the row of semi-immersed spheres considered in § 3 (*b*), and the schematic layout is shown in figure 12. The range of κd used is $2 \leqslant \kappa d \leqslant 10$, where the lower limit corresponds to the situation where the bodies touch.

The q-factors for a single row of thin ships in both beam and head seas are shown in figure 13 together with the corresponding curves for point absorbers from figure 2. Note from (2.5*b*) that the q-factor is the mean improvement or otherwise of an interacting system of N bodies upon the absorbing properties of N independent point absorbers. Thus, for an array of thin ships, $q > 1$ shows that the array works more favourably than N independent point absorbers, but not necessarily more efficiently than N independent thin ships – since the data necessary to make such a statement has not been introduced. Accordingly, when comparing bodies of different shapes, the q-factor is best regarded as a measure of maximum power absorption.

A comparison between the thin ship and point absorber curves, for either beam or head seas, shows the two curves to have essentially the same characteristics; the difference lies only in the magnitude of $q(\beta)$. In beam seas the thin ships always interact to absorb more energy than the point absorbers, whereas in head seas the reverse is true. These results would be expected from the simplest physical argument if no interaction were present: with $\kappa a = 0{\cdot}4$ and $\kappa L = 1$ each thin ship presents 2·5 times more frontage to the incident waves in beam seas than the semi-immersed spheres do and consequently has more power readily available without drawing in power from the sides. In head seas with $\kappa\epsilon = 0{\cdot}1\kappa L = 0{\cdot}1$ the spheres show four times as much frontage to the incident wave as the thin ships and can thus readily absorb more power. This argument, while not accounting for the importance of the interactions between array members, does depend crucially on directionality.

If the efficiency of the thin-ship interaction is required, then it is necessary to define the q-factor appropriate to thin ships, and which is denoted here by $Q(\beta)$.

The generalization of (2.5*b*) for any body shape is

$$l_{\max}(\beta, N) = N l_{\max}(\beta, 1)\, Q(\beta) \tag{4.2}$$

and from Evans (1979, equation (2.14)) the maximum absorption length $l_{\max}(\beta, 1)$ for a single rectangular thin ship is

$$l_{\max}(\beta, 1) = \frac{\lambda}{2\pi} M(\kappa L, \beta), \tag{4.3}$$

where $$M(\kappa L, \beta) = \left[\frac{\sin(\kappa L \sin\beta)}{\kappa L \sin\beta}\right]^2 \Bigg/ \left[\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{\sin(\kappa L\sin\theta)}{\kappa L\sin\theta}\right]^2 d\theta\right].$$

Using the definitions of $Q(\beta)$ and $q(\beta)$ from (4.2) and (2.5*b*), with the form of $l_{\max}(\beta,1)$ given by (4.3), it is readily seen that $Q(\beta)$ and $q(\beta)$ are related by

$$q_t(\beta) = M(\kappa L, \beta)\, Q_t(\beta), \tag{4.4}$$

where the suffix t denotes the 'thin-ship' result. The interactive effect of N thin ships compared to N independent thin ships is favourable whenever

$$Q_t(\beta) > 1 \quad \text{or} \quad q_t(\beta) > M(\kappa L, \beta),$$

which contains an angular dependence proportional to the square of the far-field angular variation. The values of $M(\kappa L, \beta)$ for beam and head seas with $\kappa L = 1$ are

$$M(1, 0) = 1{\cdot}1778, \quad M(1, \tfrac{1}{2}\pi) = 0{\cdot}8340 \tag{4.5}$$

and these are shown in figure 13 as broken lines.

If we require the relative efficiency of an array of point absorbers to an array of thin ships at optimal tuning, then it is necessary to compare $q_p(\beta)$ with $Q_t(\beta)$. Here the suffix p denotes the 'point-absorber' result. With use of (4.4) this is equivalent to comparing q_p to $q_t/M(\kappa L, \beta)$ and it can be seen from figure 13 that the pairs of $q_p(\beta)$ and $Q_t(\beta)$ curves will come even closer together. Hence we have the interesting result that the interactive efficiency of a row of point absorbers and thin ships is very similar in both beam and head seas whilst the actual amount of power absorbed depends upon the properties of the body shape. This suggests that array spacing can be decided on the basis of results for a simple system such as point absorbers, whilst body geometry effects can be explored by considering a simple body in isolation.

The body displacements at optimal tuning could be presented as in figures 4 and 5 for the semi-immersed spheres and then if required we could perform the constrained optimization procedure illustrated in figure 6. However, using the guide that the more power the array absorbs the larger the individual displacements, we can restrict attention to a comparison between the maximum and constrained absorption lengths. The procedure for obtaining the absorption length subject to constraints on the individual body displacement amplitudes has been described previously in § 3(*b*).

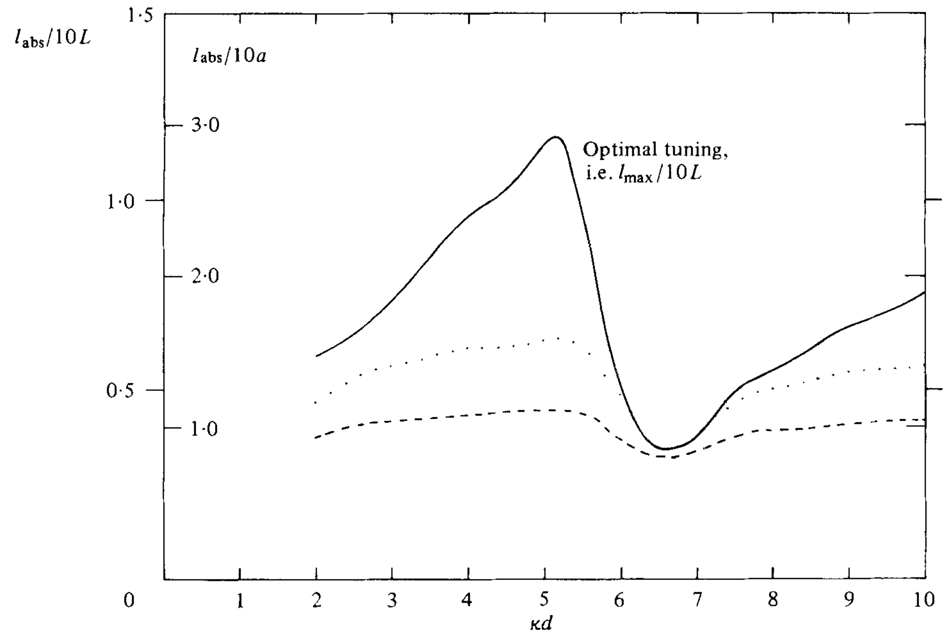

FIGURE 14. Variation of the non-dimensional absorption length $l_{abs}/10L$ with body spacing for the single row of five thin ships in beam seas. ——, optimal unconstrained motion (i.e. $l_{max}/10L$); . . . , constrained motion, maximum body displacement < three times incident wave amplitude; – – –, constrained motion, maximum body displacement < twice incident wave amplitude. The scale $l_{abs}/10a$ is also shown to enable comparison with figure 6.

The maximum and constrained absorption lengths, $l_{max}/10L$ and $l_{abs}/10L$, are plotted against κd for beam seas in figure 14, with the displacement amplitudes being constrained to be less than two and three times the incident wave amplitude. The absorption lengths are non-dimensionalized with $10L$, since $10L$ is the total frontage of the array to the incident waves. If a comparison with figure 6 is to be made then the scale for $l_{abs}/10a$ must be known and this is also shown in figure 14.

Comparison of $l_{abs}/10a$ in figures 6 and 14 shows that the constraints have a more severe effect on the performance of the thin ships than on that of the semi-immersed spheres and the implication (confirmed by calculations) is that the thin ships undergo larger displacements at optimal tuning than the semi-immersed spheres. Similar results are found for head seas, but these are not illustrated here. It is important to note that the individual body displacements are dependent upon the body geometry and for the thin ships with $\kappa L = 1$ the displacements will vary for differing values of $\kappa\epsilon$ and κH, consequently changing the constrained curves, while the curve corresponding to optimal tuning remains unchanged (though the corresponding body displacements may change). Our results for constrained motion apply only to the case of $\kappa L = \kappa H = 1$ and $\kappa\epsilon = 0{\cdot}1$.

For both beam and head seas the body displacement amplitudes at optimal tuning are symmetric about the midpoint of the row. The result in fact holds at any arbitrary angle of incidence for any optimally tuned uniform row of similarly aligned identical devices which possess two vertical mutually perpendicular planes of symmetry, provided the assumptions made here concerning the diffracted wave field remain valid.

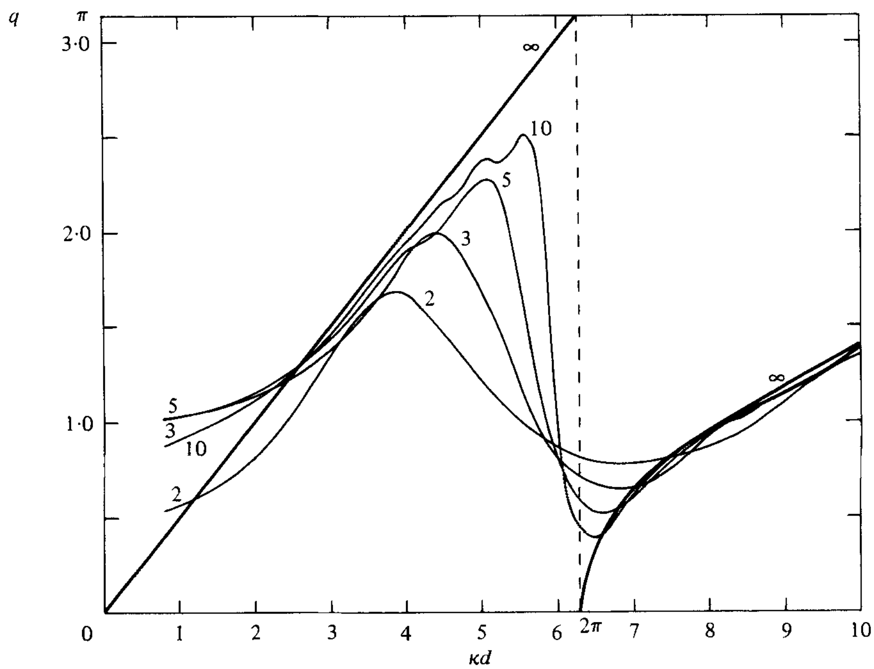

FIGURE 15. Variation of the mean gain factor q with non-dimensional body spacing κd for optimally tuned equally spaced single rows of two, three, five and ten point absorbers in beam seas. The heavy solid line corresponds to the infinite array limit given for $\kappa d < 2\pi$ by Evans (1979) and for general κd by Srokosz (1980).

This general result follows directly from the same arguments as those presented by Evans (1979).

5. Discussion

The results given in this paper are for five uniformly spaced identical bodies (or two parallel such rows) in beam and head seas. The calculations were made for rows comprised of two, three, five and ten bodies, for both the optimally tuned and constrained motions of the semi-immersed spheres and 'thin ships', with the three angles of incidence $\beta = 0$ (beam seas), $\frac{1}{4}\pi$, $\frac{1}{2}\pi$ (head seas). Clearly it is not possible to present all of the available data and it is felt that the important features of the interactions can be demonstrated by choosing five bodies (or ten, in the case of the double row) in beam and head seas. These two angles of incidence provide data which is relatively easy to interpret, and furthermore they provide possible models for two classes of wave-energy devices, terminators and attenuators, which are designed to operate efficiently in beam and head seas respectively.

The choice of five bodies was made after study of all the data and was also guided by the need to model possible physical array configurations. As an example of the variation of the optimal array properties with the number of bodies we present in figure 15 the behaviour of the mean gain factor q with body spacing κd for two, three, five and ten point absorbers in beam seas. The heavy solid line corresponds to the infinite array limit; this was first given for $\kappa d < 2\pi$ by Evans (1979) and for all values of κd by Srokosz (1980). Note that, from the infinite array analysis of Srokosz, all bodies have the same displacement amplitude at optimal tuning; for five bodies this is not so (as shown in figure 3) and is directly attributable to the array being of finite length.

6. Conclusions

The maximum amount of wave power that can be absorbed by both a single and double row of five vertically oscillating floating bodies in a regular incident wave train has been estimated. The bodies chosen were semi-immersed spheres in one case and thin wedge-shaped bodies in the other. The simple shapes chosen enabled explicit analytical forms to be derived for the maximum absorbed power when the waves approach at any angle.

It was found that the improvement in maximum power absorption of a given array over a single member of the array in isolation was influenced primarily by the spacing and not by the particular body geometry. The latter governed the actual power absorbed, being larger for the elongated thin bodies in beam seas and less in head seas than for the sphere. This reflects the relative wave-making ability of the thin body perpendicular to and in the direction of its axis. When constraints were imposed on the amplitude of the body displacements, the thin body suffered a greater drop in power absorption than the sphere; this being due to the poor wavemaking capability of the 'thin ship' in heave compared to the sphere.

The results of this paper suggest that the spacing between members of an array of bodies should be just less than a wavelength to achieve maximum absorption in beam seas and that this conclusion is largely independent of the shape of the body. It is anticipated that suitably spaced elongated bodies absorbing energy through horizontal motions in the direction of the incident waves, will achieve a further improvement in maximum energy absorption even when constrained, provided they have good wave-making characteristics.

However, it must be emphasized that the above conclusions are valid only for arrays which satisfy the approximations made in this paper, that is, for arrays where the wave field created by the motion of any one member is not influenced by the presence of the remaining bodies. Further work is essential to determine the exciting force on an array of full-bodied elongated absorbers to assess their potential wave-absorbing capacity.

Appendix

Maximum power absorption by an array of bodies

We consider N independently oscillating bodies each capable of absorbing wave energy through its motion in a single degree of freedom. The ith component F_{is} of the exciting force vector $\mathbf{F}_s$ is the force, in the direction of subsequent motion, on the ith body assuming all bodies are held fixed. Under the assumption of simple harmonic motion of radian frequency ω, we write $\mathbf{F}_s = \mathrm{Re}\{\mathbf{X}_s e^{i\omega t}\}$, and $\mathbf{U} = \mathrm{Re}\{\mathbf{U}_0 e^{i\omega t}\}$, where $\mathbf{U}$ is the velocity vector whose ith component is the velocity of the ith body. $\mathbf{M}$ and $\mathbf{B}$ are the generalized added mass and damping matrices for the system which contribute to the total hydrodynamic force vector for the system.

Thus $\mathbf{F}(t) = \mathbf{F}_s(t) - \mathbf{M}\dot{\mathbf{U}}(t) - \mathbf{B}\mathbf{U}(t)$. The mean rate of working of the hydrodynamic forces on all the bodies is

$$P = \overline{\mathbf{F}^T(t)\,\mathbf{U}(t)}^t = \overline{\mathbf{F}_s^T(t)\,\mathbf{U}(t)}^t - \overline{\mathbf{U}^T\mathbf{B}\mathbf{U}}^t,$$

where —t denotes time average and T denotes the transpose.

Hence
$$\begin{aligned} P &= \tfrac{1}{2}\,\mathrm{Re}\,\{\mathbf{X}_s^*\,\mathbf{U}_0\} - \tfrac{1}{2}\mathbf{U}_0^*\,\mathbf{B}\mathbf{U}_0 \\ &= \tfrac{1}{8}\mathbf{X}_s^*\,\mathbf{B}^{-1}\mathbf{X}_s - \tfrac{1}{2}(\mathbf{U}_0 - \tfrac{1}{2}\mathbf{B}^{-1}\mathbf{X}_s)^*\,\mathbf{B}(\mathbf{U}_0 - \tfrac{1}{2}\mathbf{B}^{-1}\mathbf{X}_s)\end{aligned}$$

after some elementary manipulation, in agreement with (2.1).

The relationship between the damping matrix and the exciting forces

For the system of bodies described above in (i), the potential function Φ is usually written as the sum of potentials describing the diffractive and radiative components of the motion

$$\Phi = \mathrm{Re}\left\{\left[(\phi_0 + \phi_d) + i\omega \sum_{j=1}^{N} \xi_j \phi_j\right] e^{i\omega t}\right\},$$

where $\phi_0 + \phi_d$ is the scattering potential satisfying $\partial(\phi_0 + \phi_d)/\partial n = 0$ on each of the bodies. The radiation potentials ϕ_j satisfy $\partial\phi_j/\partial n = n_j\,\delta_{j\kappa}$ for $\kappa = 1, \ldots, N$ and hence only contribute on the jth body; the quantity ξ_j being the displacement of the jth body.

The real generalized added mass and damping matrices, $\mathbf{M}$ and $\mathbf{B}$, depend only on the radiation potentials and are given by

$$\omega^2 M_{ij} - i\omega B_{ij} = -\rho\omega^2 \int_{S_i} \phi_j \frac{\partial \phi_i}{\partial n}\, ds.$$

Applying Green's theorem to $\bar{\phi}_i$ and ϕ_j, together with the symmetry properties of M_{ij} and B_{ij}, enables B_{ij} to be determined in the form

$$B_{ij} = \frac{\rho\omega}{2i}\int_{S_R}\left[\bar{\phi}_i \frac{\partial\phi_j}{\partial n} - \phi_j \frac{\partial\bar{\phi}_i}{\partial n}\right] ds,$$

where S_R is taken to represent a large vertical circular cylinder containing the array. The asymptotic behaviour of the potentials is known to be $\phi_i \sim g_i(\theta)\,R^{-\frac{1}{2}}\,e^{-i\kappa R + \kappa Z}$ and hence

$$B_{ij} = \frac{\rho\omega}{2}\int_0^{2\pi} \bar{g}_i(\theta)\,g_j(\theta)\,d\theta. \tag{A 1}$$

The exciting force on the ith body $F_{si} = \mathrm{Re}\,\{X_{si}\,e^{i\omega t}\} = \mathrm{Re}\left\{i\omega\rho\int_{S_i}(\phi_0 + \phi_d)\,n_i\,ds\right\}$, where ϕ_0 is the incident wave train ($\phi_0 = -iAg\omega^{-1}\exp\,[i\kappa(x\cos\beta + y\sin\beta) + \kappa z]$ for an arbitrary angle of incidence β) and ϕ_d is the diffractive field due to the bodies. The dependence of X_{si} upon ϕ_d can be removed by applying Green's theorem to ϕ_d and ϕ_i, together with the boundary conditions on each of the bodies, to give

$$\begin{aligned} X_{si} &= i\omega\rho \sum_{\kappa=1}^{N}\int_S \left[\phi_0 \frac{\partial\phi_i}{\partial n} - \phi_i\frac{\partial\phi_0}{\partial n}\right] ds \\ &= i\omega\rho\int_{S_R}\left[\phi_0\frac{\partial\phi_i}{\partial n} - \phi_i\frac{\partial\phi_0}{\partial n}\right] ds\end{aligned}$$

after a further application of Green's theorem to ϕ_0 and ϕ_i. The behaviour of ϕ_0 and ϕ_i is known on S_R, enabling the integral to be evaluated using the method of stationary phase,

$$X_{si} = \rho g A\left(\frac{2\pi}{\kappa}\right)^{\frac{1}{2}} g_i(\beta)\, e^{-\frac{3}{4}\pi}. \tag{A 2}$$

Substitution for $g_i(\beta)$ into the expression (A 1) for the damping matrix then yields expression (2.2).

The heave exciting force on a 'thin ship'

The derivation of (4.1) proceeds as follows. The thin ships are equally spaced in a line along the y axis, the nth being centred at L_n: $x = 0,\ -H \leqslant z \leqslant 0, nd-L \leqslant y \leqslant nd+L$. It is symmetric about $x = 0$, being described by $x = \xi(y,z) = \epsilon(H+z)H$.

The incident wave has elevation $Ae^{\kappa z}\cos(\kappa x\cos\beta + \kappa y \sin\beta + \omega t)$ and a corresponding velocity potential $\mathrm{Re}\{\phi_0 e^{i\omega t}\}$ with $\phi_0 = -iAg\omega^{-1}\exp[i\kappa(x\cos\beta + y\sin\beta) + \kappa z]$.

Since the body is thin the condition of zero normal velocity can be applied on L_n and the heave exciting force on the nth body is

$$X_{sn}(\beta) = -i\omega\rho \iint (\phi_0 + \phi_d)\frac{\partial \xi}{\partial z}\, ds,$$

where the integration is over both sides of the ship. Now the diffracted potential ϕ_d is odd in x and hence does not contribute to $X_{sn}(\beta)$. Taking into account both sides of the ship, we get

$$\begin{aligned} X_{sn}(\beta) &= -\frac{2i\omega\rho\epsilon}{H}\iint_{L_n} \phi_0\, dz\, dy \\ &= -\frac{4\rho g A\epsilon L(1-e^{-\kappa H})}{\kappa H}\frac{\sin(\kappa L\sin\beta)}{\kappa L \sin\beta}\, e^{i\kappa dn \sin\beta} \end{aligned}$$

in agreement with (4.1).

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